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A novel method to determine laminar convective heat transfer in the entry region of a tube

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Abstract

A new method is suggested to estimate the average heat transfer coefficients in the entry region of a short length of tube under developing laminar flow conditions. The method, though appears to be unconventional, makes use of the concept that the turbulent mixing length theory can be extrapolated down to the laminar region with the universal constant K in the eddy diffusivity expression to be dependent on L/D and Reynolds number. However, in the estimation of the mean heat transfer coefficients in the entry region, the friction coefficients data must be known *a priori*. Comparison of the present theory with the correlations of Hausen [Z. VDI Beihefte Verfahrenstechnik 4 (1943) 91] and Seider and Tate [Ind. Engrg. Chem. 28 (1936) 1429] related to entry region analysis revealed satisfactory agreement substantiating the validity of the approach.

Keywords: Entry region; Eddy diffusivity; Average friction coefficient; Mean heat transfer coefficient

1. Introduction

Convective heat transfer studies in tubes have been thoroughly investigated by many and information is made available in the literature and handbooks [1] in the forms of both correlations and theory for developing hydrodynamic and thermal boundary conditions. Analysis of convective heat transfer in short lengths of tubes in laminar flow region is very much pertinent to the compact heat exchanger design [2]. Hausen [3] compared various correlations available in literature in a single plot even though the thermal conditions prescribed at the wall are different in nature. It can be seen that the trends are similar and the orders of magnitude more or less the same. However, the average asymptotic Nusselt numbers are 4.32 and 3.63, respectively, for constant wall temperature and constant heat flux conditions. Because of the developing nature of the boundary layer in the entry region, the flow essentially becomes two-dimensional with the radial component of velocity assuming the same order of magnitude as the axial component. Consequently the mixing length concept can be extended in the estimation of heat transfer coefficient down to the laminar region from the turbulent flow condition by treating one of the constants in the eddy diffusivity expression to be dependent on the hydrodynamic conditions and other geometric parameters of the tube.

Thus, the purpose of this article is to predict the mean heat transfer coefficient for the entry region of a tube using the mixing-length concept with the momentum transfer characteristics prescribed as input. It is observed that the mean heat transfer coefficients predicted in the present study for a given Pr, (L/D) and Re combination through Gz, Gratz number agree reasonably well with those of equations of Hausan [3] and Seider and Tate [4] for constant heat flux and constant wall temperatures, respectively. The rapid change in hydrodynamic and thermal conditions in the entry region of short length of a tube leads to substantial increase in wall friction coefficients and average heat transfer coefficients. The correlation of Seider and Tate is for high Prandtl viscous fluids. Thus, the purpose of the present article is to demonstrate a modification of eddy diffusivity ε_m by changing one of the parameters K or A^+ in the expression of van Driest [5], which would respond favorably to laminar region as well.

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Nomenclature

A^+	damping constant in Eq. (1)	
C_p	specific heat	$J{\cdot}kg^{-1}{\cdot}K^{-1}$
D	diameter of the tube	m
D^+	dimensionless tube diameter, $= Du^*/\nu$	
f	friction coefficient	
h	heat transfer coefficient	$W{\cdot}m^{-2}{\cdot}K^{-1}$
k	thermal conductivity	$W{\cdot}m^{-1}{\cdot}K^{-1}$
L	length of the tube	m
$\dot{m}$	discharge rate of the fluid	$kg{\cdot}s^{-1}$
Nu_m	mean Nusselt number, $= h_m D/k$	
Pr	Prandtl number, $= \mu C_p/k$	
R	radius of the tube	m
Re	Reynolds number, $= \frac{4\dot{m}}{\pi D\mu}$	
R^+	dimensionless radius	
T	temperature	
T^+	dimensionless temperature	
u	velocity	$m{\cdot}s^{-1}$
u^*	shear velocity	$m{\cdot}s^{-1}$
u^+	dimensionless velocity, $= u/u^*$	
x	distance measured from the entrance to the tube	m
y	distance measured normal to the wall	m
y^+	dimensionless distance measured normal to the wall, $= yu^*/\nu$	

Greek symbols

$\varepsilon_h, \varepsilon_m$	thermal and momentum eddy viscosities	$m^2{\cdot}s^{-1}$
τ	shear stress	$N{\cdot}m^{-2}$
μ	absolute viscosity	$kg{\cdot}m^{-1}{\cdot}s^{-1}$
η	dimensionless distance, $= y^+/R^+$	
ν	kinematic viscosity	$m^2{\cdot}s^{-1}$
ρ	density	$kg{\cdot}m^{-3}$

Subscripts

B	bulk
c	centre line
m	mean
w	wall

2. Formulation

The problem is formulated with the following assumptions.

- The eddy diffusivity expression as postulated by Sarma et al. [6] is considered in the analysis. The relationship is a modification of van Driest's expression [5]. It is assumed that the constants K and A^+ appearing in the expression are respectively 0.4 and 23 for single phase turbulent convective heat transfer

$$\frac{\varepsilon_m}{\nu} = K y^+\big[1 - \exp\big(-y^+/A^+\big)\big]^2 \tag{1}$$

In Eq. (1) for laminar region ε_m should tend to negligible value. It follows that either $K \to 0$ or $A^+ \to \infty$. The damping constant, $A^+ = 23$ is takenin the present study. K is considered to be dependent on L/D and Re.

- For axisymmetric flows in tubes, the shear stress distribution is given by

$$\frac{\tau}{\tau_w} = 1 - \frac{y^+}{R^+} \tag{2}$$

where

$$R^+ = \frac{Ru^*}{\nu} = \frac{Ru_m}{\nu}\sqrt{\frac{f}{2}} = \frac{Re}{2}\sqrt{\frac{f}{2}}$$
$$\tau_w = \frac{1}{2} f\rho u_m^2 \quad \text{and} \quad \rho u_m\left[\pi\frac{D^2}{4}\right] = \dot{m} \tag{2a}$$

- Physical property variation with respect to spatial temperature variation is ignored in the analysis, though it is pretty well known that velocity profiles are profoundly influenced by the temperature dependent properties. Further the contribution of convective terms is assumed to be taken care indirectly by the eddy diffusivity parameter ε_m/ν to be evaluated satisfying the momentum transfer characteristic viz., the mean friction coefficient, f, i.e., Eq. (3).
- The thermal eddy diffusivity ε_h is equal to ε_m for $Pr \geqslant 1$.
- The asymptotic friction coefficient for fully developed laminar region is given by $f_{L/D=\infty} = 16/Re$ where $Re = (4\dot{m}/\pi D\mu)$. However, the average friction coefficient f for a short length L of the tube, is given by the following expression [7]

$$f = \big[0.312(D/L) + 16/Re\big]\tanh\big[5.02\big(L/(D\,Re)\big)^{1/2}\big] \tag{3}$$

Satisfying the asymptotic conditions, i.e., when $D/L \to 0$, $f = 16/Re$.

This equation holds good for hydrodynamically developing region of the tube (i.e., the entrance region). Thus, the assumptions would enable us to check the applicability of the concept to the laminar flows in the evaluation of average heat transfer coefficients for a given L/D.

Analysis

The configuration under consideration is a short tube of finite length L, in which the flow is of developing nature. The boundary layer development along the flow direction occurs in the flow direction. Subject to certain assumptions the average heat transfer coefficients will be evaluated for given L/D.

Thus, the mass rate of flow $\dot{m}$ across the tube is given by the realtionship

$$\dot{m} = \int_0^R \rho u 2\pi (R-y)\,\mathrm{d}y \tag{4}$$

Or in the dimensionless form

$$\frac{Re}{4} = \int_0^{R^+} u^+\left[1 - y^+/R^+\right]\mathrm{d}y^+ \tag{5}$$

For isothermal conditions excluding the curvature effects the velocity profile in the stream can be obtained from the following relationship as per the assumptions (see Eq. (2)).

$$\frac{\mathrm{d}u^+}{\mathrm{d}y^+} = \left(\frac{1 - y^+/R^+}{1+\varepsilon_m/\nu}\right) \tag{6}$$

The boundary condition is $u^+ = 0$ at $y^+ = 0$.

Besides, ignoring the curvature effects the temperature profile can be evaluated on the assumption that the radial turbulent conduction is the dominant mode.

$$\frac{\mathrm{d}}{\mathrm{d}y^+}\left[\left(1 + \frac{\varepsilon_m}{\nu}Pr\right)\frac{\mathrm{d}T^+}{\mathrm{d}y^+}\right] = 0 \tag{7}$$

where $T^+ = \frac{T_w - T}{T_w - T_c}$ and T_c is the centre line temperature.

Eq. (7) is the result of the assumption that convection can be partially accommodated if not totally through the unknown parameter ε_m/ν which is to be iterated satisfying the mean friction coefficient characterstic.

The boundary conditions are

$$\begin{aligned} &\text{at } y^+ = 0, \quad T^+ = 0 \\ &\text{at } y^+ = R^+, \quad T^+ = 1 \end{aligned} \tag{8}$$

Thus, Eqs. (3)–(8) would enable us to find the mean convective heat transfer coefficient as follows.

3. Local and mean heat transfer coefficients

The heat flux at the wall is given by

$$q_w = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = h_x(T_w - T_B) \tag{9}$$

The mean heat transfer coefficient h_m can be evaluated from the condition

$$h_m = \frac{1}{L}\int_0^L h_x\,\mathrm{d}x$$

$$h_m = \frac{1}{L}\int_0^L \frac{1}{T_w - T_B} q_w\,\mathrm{d}x = -\frac{1}{L}\int_0^L \frac{k}{T_w - T_B}\frac{\partial T}{\partial y}\bigg|_{y=0}\mathrm{d}x$$

It can be seen that $h_m = h_x$ if the variations are arbitrarily chosen as $q_w = c_1 x^n$ and $(T_w - T_B) = c_2 x^n$ where n is index.

Imposing a thermal constraint as an assumption that the ratio of local flux to the local value of $(T_W - T_B)$ is constant and independent of flow direction over the length L of the tube it follows that Eq. (9) in dimensionless form can be expressed as follows:

$$\frac{h_m D}{k} = D^+\frac{\partial T^+}{\partial y^+}\bigg|_{y^+=0}\phi \tag{10}$$

where

$$D^+ = 2R^+ = Re\sqrt{\frac{f}{2}}$$

$$\phi = \frac{T_w - T_c}{T_w - T_B}$$

and

$$\frac{1}{\phi} = \frac{T_w - T_B}{T_w - T_c} = \frac{\int_0^{R^+} u^+ T^+[1 - y^+/R^+]\,\mathrm{d}y^+}{\int_0^{R+} u^+[1 - y^+/R^+]\,\mathrm{d}y^+}$$

The temperature ratio term ϕ can be obtained when once the velocity and temperature profiles are known and its value is defined by Eq. (11). From the formulation so far presented one would get the impression that it is related to the fully developed turbulent convective heat transfer problem. It can be seen that there is underlying generality in the formulation and when $\varepsilon_m/\nu = 0$ (i.e., $K \to 0$ in the eddy diffusivity expression) the solution corresponds to the fully developed laminar flow in a tube. Thus the solution for the case of the laminar convective heat transfer is inherently embedded in the formulation. This asymptotic value can be obtained by putting $\varepsilon_m/\nu = 0$ in Eqs. (6) and (7) and solving for velocity and temperature profiles as follows.

$$u^+ = y^+ - y^{+2}/2R^+ \tag{11}$$

$$T^+ = y^+/R^+ \tag{12}$$

$$\phi = 15/7 \tag{13}$$

From Eq. (10) with the help of Eqs. (11)–(13), it can be obtained that the asymptotic Nusselt number is

$$Nu = 2 \times 15/7 = 4.29 \tag{14}$$

The asymptotic value is specific to the thermal conditions and assumptions imposed on the problem. Consequently, the magnitude of (ε_m/ν) between zero and the one corresponding to fully developed turbulent flow conditions might be construed as a different flow situation. In particular, in the present study the average friction coefficient in short lengths is taken as the hydrodynamic constraint to predict the average heat transfer coefficients for under-developed laminar flow in short lengths of tubes. Thus, the following numerical procedure is employed to investigate the average heat transfer coefficients in hydrodynamically and thermally developing regions.

- Assume the input data, i.e., Pr, R^+ and (L/D).
- Solve Eq. (6) for velocity profile for an assumed value of K in the eddy diffusivity equation (1).
- Estimate the value of Re from Eq. (5).
- Estimate the average value of the friction coefficient 'f' from Eq. (2a), i.e.,

$$f = 8\left[\frac{R^+}{Re}\right]^2 \tag{15}$$

- Compare this value of 'f' from the value obtained from Eq. (3). If this difference between the two is less than 10^{-7}, the assigned value to K in the eddy diffusivity expression is correct or otherwise an appropriate iteration technique is employed such that for a given values of Pr, R^+, L/D magnitudes of Re, f and ε_m are determined.
- Assuming $\varepsilon_m = \varepsilon_h$ the temperature profile is further determined making use of Eq. (7) subject to the boundary conditions, i.e., Eq. (8).
- Eq. (10) fixes the required value of the mean heat transfer coefficient for the given input conditions.
  Thus, $Nu_m = F[Re, L/D, Pr]$.

The numerical procedure is accomplished for the ranges as follows.

$$5 < L/D < 200, \qquad 1 < Pr < 100, \qquad 100 < Re < 2300$$

4. Validation of the method

Hausen [3] developed a correlation for average Nusselt number in the entry region as follows.

$$Nu_m = 3.65 + \frac{0.0668\frac{D}{L}Re\,Pr}{1 + 0.04[\frac{D}{L}Re\,Pr]^{2/3}} \tag{16}$$

The equation of Seider and Tate [4] for developing conditions of the flow is

$$Nu_m = 1.86(Re\,Pr\,D/L)^{1/3} \tag{17}$$

These two correlations are for two different thermal conditions but yet the salient feature in the two analyses are that the entry region convective mean heat transfer is dependent on a unique parameter known as Gratz number a combination of geometric conditions with other hydrodynamic and thermal π groupings. These equations are often referred to in the heat transfer literature. The present analysis is shown plotted in a conventional manner for $L/D = 5, 10, 30$ and $Pr = 1, 100$. It can be seen that the results of the present theory very satisfactorily agree for the ranges shown in the Fig. 1 with the correlation equations of the Seider and Tate [4] and Hausen [3] except for the deviation in asymptotic values. Nevertheless, the correlation of Seider and Tate has limitation on the magnitude of L/D since $Nu_m \to 0$ as $L/D \to \infty$. The analysis indicates that the concept employed reasonably gives close agreement with correlations

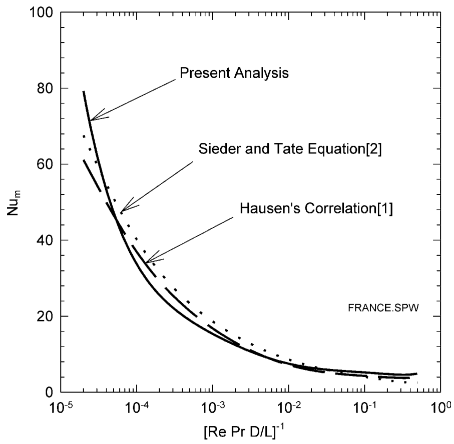

Fig. 1. Comparison of various analyses.

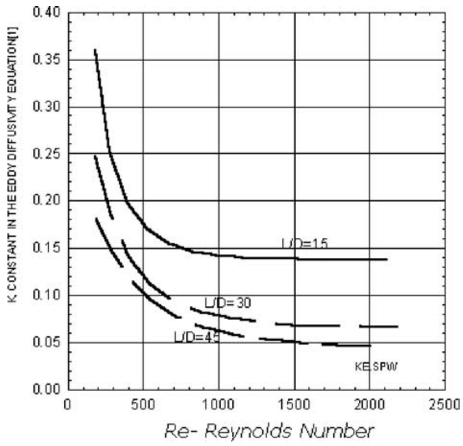

Fig. 2. Variation of K with Re.

for the range of the Prandtl numbers as well. In the developing region the flow is primarily two-dimensional since the radial component of velocity is of the same order as the axial component. Consequently, the value of K will be having higher magnitudes for low values of L/D. However, as L/D increases the flow becomes one-dimensional in nature and tends to fully developed conditions. It implies that K in Eq. (1) must tend to zero for fully developed conditions. Such trends can be seen in Fig. 2.

Within the frame work of assumptions the nature of velocity distribution across the tube for different values of $20 < L/D < 250$ is shown in Fig. 3. It can be seen that as the value of L/D increases the velocity profiles tend towards parabolic distribution represented by the curve, i.e., Eq. (11). The influence of (L/D) on temperature profiles can be noticed in Fig. 4. It can be observed that as L/D increases the temperature variation is becoming linear as given by Eq. (12). Further, justification of the approach can be visualized from the magnitudes of ε_m/ν shown in Fig. 5. Evidently as L/D increases the cross flow component of the velocity decreases and its value diminishes towards zero. It indirectly supports the fact that there is minimum length requirement for the flow to be fully developed.

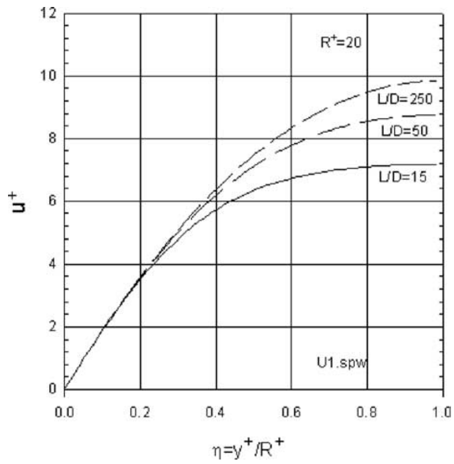

Fig. 3. Velocity profiles [$15 < L/D < 250$].

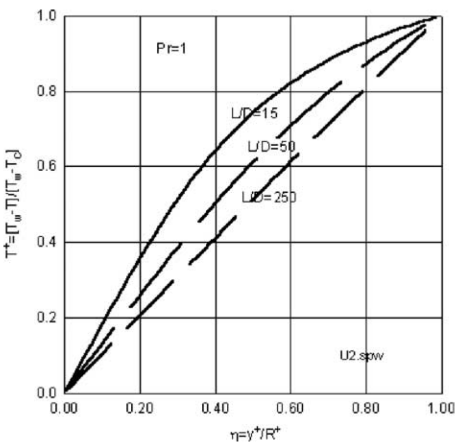

Fig. 4. Effect of L/D on temperature profiles.

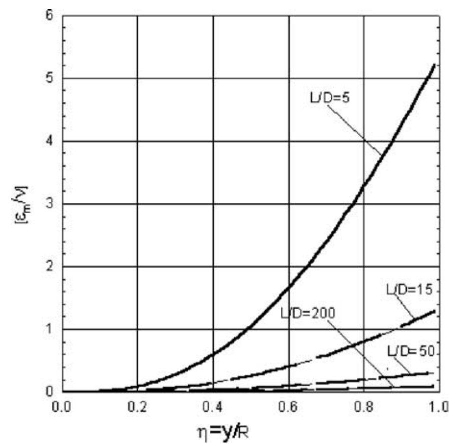

Fig. 5. Variation of eddy diffusivity.

Since the trends and magnitudes of results for mean Nusselt number are in conformity with the correlations of Hausen [3] and Sieder and Tate [4], regression is applied to 522 data points numerically obtained from the computer runs for the ranges indicated and the following equation is obtained with a standard deviation of 8.2% and average deviation of 6.7%.

$$Nu_m = 0.964\left\{\exp\left[1.278\left(Re\,Pr\,\frac{D}{L}\right)^{0.1084}\right]\right\}^{1.03} \tag{18}$$

Besides, another correlation for the prediction of the average friction coefficient is also proposed from 522 computational points from the runs. It can be considered as an alternative form of the one made use of in the study, i.e., Eq. (3).

$$\frac{f}{f_{L/D=\infty}} = 1.0341Re^{0.093}\left[\frac{L}{D}\right]^{-0.124} \tag{19}$$

where $f_{L/D=\infty} = 16/Re$.

5. Conclusions

Thus, the following conclusions can be made from the study of convective heat transfer in short length tubes.

It is found that the turbulent theory can be extrapolated into the laminar region by considering constant K in the eddy diffusivity relationship as a dependent parameter on $[L/D\&Re]$. Though the approach is non-conventional in predicting the mean convective heat transfer, the friction data from momentum transfer studies must be known *a priori*. In other words the Colburn's analogy can be implemented successfully even for a complex situation of the entry region with the aid of the concept that the eddy diffusivity relationship for turbulent flows can be reduced to the special case of laminar flow by considering the constant $K = F[Re, L/D]$. Hence a modification of Prandtl's mixing length theory is applicable to the entry region as well. Thus, Eqs. (18) and (19) are the correlations to evaluate mean heat transfer and friction coefficients.

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